# NOTES FROM YESTERDAY:

## Vector Component Form

*given* : $(x_1, y_1)$

$(x_2, y_2)$

$$v = \langle x_2 - x_1, \; y_2 - y_1 \rangle$$

*horizontal ↑ and ↑ vertical components*

## Magnitude

$$|v| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

or $|v| = \sqrt{x^2 + y^2}$

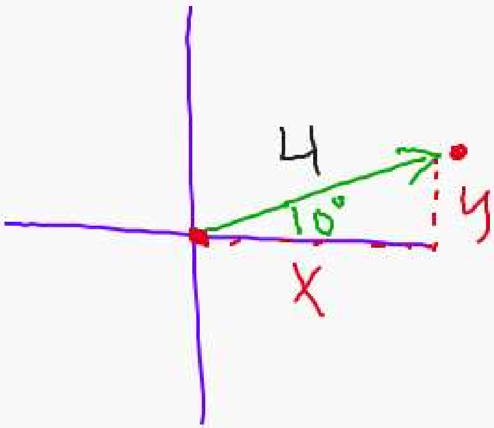
*horizontal vertical components*

↑*This is the distance formula... a form of the Pythagorean Theorem.*

**41–46 ■ Components of a Vector** Find the horizontal and vertical components of the vector with given length and direction, and write the vector in terms of the vectors **i** and **j**.

Use degree mode!

## 9.1 #45 $|\mathbf{v}| = 4, \quad \theta = 10°$




$\cos 10° = \frac{x}{4}$

$x = 4\cos 10°$

$x \approx 3.94$

$\sin 10 = \frac{y}{4}$

$y = 4\sin 10°$

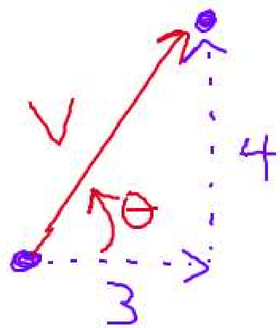
$y \approx 0.69$

$\boxed{v = 3.94i + 0.69j}$

**47–52 ■ Magnitude and Direction of a Vector** Find the magnitude and direction (in degrees) of the vector.

**9.1 #47** $\mathbf{v} = \langle 3, 4 \rangle$

horiz vert




$$|V| = \sqrt{3^2 + 4^2}$$

$$= \sqrt{25}$$

$$\boxed{|V| = 5}$$

magnitude

$$\tan\theta = \frac{4}{3}$$

$$\theta = \tan^{-1}\left(\frac{4}{3}\right)$$

$$\boxed{\theta \approx 53.13^\circ}$$

direction

## DEFINITION OF THE DOT PRODUCT

**NOTES 9.2**

If $\mathbf{u} = \langle a_1, a_2 \rangle$ and $\mathbf{v} = \langle b_1, b_2 \rangle$ are vectors, then their **dot product**, denoted by $\mathbf{u} \cdot \mathbf{v}$, is defined by

multiply like components

$$\mathbf{u} \cdot \mathbf{v} = a_1 b_1 + a_2 b_2$$

**The dot product is not a vector; it is a real number, or scalar (comparison of slopes.)**

**If u•v = 0, then vector u and v are perpendicular.**

NOTES 9.2

## ANGLE BETWEEN TWO VECTORS

If $\theta$ is the angle between two nonzero vectors $\mathbf{u}$ and $\mathbf{v}$, then

$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{u}|\,|\mathbf{v}|}$$

← dot product

Magnitude →

← Magnitude

**5–14 ■ Dot Products and Angles Between Vectors** Find **(a) u · v** and **(b) the angle** between **u** and **v** to the nearest degree.

**8.** $\mathbf{u} = \langle -6, 6 \rangle, \quad \mathbf{v} = \langle 1, -1 \rangle$

$$|u| = \sqrt{(-6)^2 + 6^2} = \sqrt{72} = 6\sqrt{2}$$

$$|v| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$$

a. $u \cdot v = -6(1) + 6(-1)$

$= -6 - 6$

$u \cdot v = \boxed{-12}$ ✓

b. $\cos\theta = \dfrac{u \cdot v}{|u||v|} = \dfrac{-12}{6\sqrt{2}\sqrt{2}}$

$\cos\theta = \dfrac{-12}{6 \cdot 2}$

$\cos\theta = \dfrac{-12}{12}$

$\cos\theta = -1$

$\boxed{\theta = 180°}$ from unit circle

*use degree mode!!
*use exact values from unit circle when possible

CHECK ALL EVEN & ODD ANSWERS:

<u>**9.2 #8-20**</u>

-12 -10 -4 -1 0 0 0 0

0 1 4 4 $5\sqrt{3}$ 30 60.26

85.60 90 97.13 153.43 180

perpendicular not perpendicular
perpendicular not perpendicular
perpendicular
perpendicular

**5–14 ■ Dot Products and Angles Between Vectors** Find (a) $\mathbf{u} \cdot \mathbf{v}$ and (b) the angle between $\mathbf{u}$ and $\mathbf{v}$ to the nearest degree.

or $3i - 2j$ $\qquad$ $i + 2j$

**9.** $\mathbf{u} = \langle 3, -2 \rangle, \quad \mathbf{v} = \langle 1, 2 \rangle$

a. $u \cdot v = 3(1) + -2(2)$

$= 3 - 4$

$\boxed{u \cdot v = -1}$

$|u| = \sqrt{3^2 + (-2)^2}$

$= \sqrt{13}$

$|v| = \sqrt{1^2 + 2^2}$

$= \sqrt{5}$

b. $\cos\theta = \dfrac{u \cdot v}{|u||v|} = \dfrac{-1}{\sqrt{13}\sqrt{5}}$

$\cos\theta = \dfrac{-1}{\sqrt{65}}$ not on unit circle

$\theta = \cos^{-1}\left(\dfrac{-1}{\sqrt{65}}\right)$ use calculator $\rightarrow \boxed{\theta \approx 97.13^\circ}$

***use degree mode!!**
***use exact values from unit circle when possible**

CHECK ALL EVEN & ODD ANSWERS:

**9.2 #8-20**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| -12 | -10 | -4 | -1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 4 | 4 | $5\sqrt{3}$ | 30 | 60.26 | |
| 85.60 | 90 | 97.13 | 153.43 | 180 | | | |

| | |
|---|---|
| perpendicular | not perpendicular |
| perpendicular | not perpendicular |
| perpendicular | |
| perpendicular | |

**5–14 ■ Dot Products and Angles Between Vectors** Find (a) $\mathbf{u} \cdot \mathbf{v}$ and (b) the angle between $\mathbf{u}$ and $\mathbf{v}$ to the nearest degree.

**10.** $\mathbf{u} = 2\mathbf{i} + \mathbf{j}, \quad \mathbf{v} = 3\mathbf{i} - 2\mathbf{j}$

**same as** $\rightarrow u = \langle 2,1 \rangle \quad v = \langle 3,-2 \rangle$

***use degree mode!!**
***use exact values from unit circle when possible**

CHECK ALL EVEN & ODD ANSWERS:

**9.2 #8-20**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| -12 | -10 | -4 | -1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 4 | 4 | $5\sqrt{3}$ | 30 | 60.26 | |
| 85.60 | 90 | 97.13 | 153.43 | 180 | | | |

| | |
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